easy by this method, and so well adapted to practice, that operatives of every class will find it greatly to their advantage to employ it.

Having thus exhibited a simple and practical method of describing the pentagon, we have now to shew the method of its conversion; first, into a trapezium; second, into a triangle; third, into a rectangle; fourth, into a square; and fifth, into a circle. The figure by which all these conversions are represented must necessarily be very complicated, and might have been separated into several subordinate parts with considerable advantage as regards distinctness; but wishing as much as possible to avoid a multiplicity of diagrams, the whole is condensed into one figure, as under.

Let ABCDE (fig. 2.) be the given regular pentagon, of which the sides are AB, BC, CD, DE, and EA respectively, and which it is required to convert successively into the several figures specified above.

Draw the diagonals AD and BD, dividing the given pentagon into the three isosceles triangles ADE, ADB, and BDC, of which ADE and BDC are equal and similar in every respect. Extend the side BC directly forward to N, making CN equal to CB or CD, the side of the pentagon, and draw DN; then is the triangle CDN equal to the triangle BDC, or ADE; and consequently the trapezium ABND is equal to the pentagon ABCDE; for the triangle CDN, which is added to the pentagon on the side CD, is equal to the triangle ADE, which is separated from it by the diagonal AD.

Produce CB and EA, the sides of the given pentagon, to meet each other in the point F, and draw DF; then is the triangle FDN equal to the pentagon ABCDE, for it is equal to the trapezium ABND, and it has just been shewn that the trapezium and pentagon are equal, so also are the triangle and pentagon.

Bisect the diagonal BD perpendicularly in the point Z by the straight line CG, and from the point N let fall the perpendicular NO, meeting GC produced in O, and complete the rectangular parallelogram GHNO, which will also be equal to the given pentagon ABCDE; for it is obviously equal to the trapezium ABND, and it has been shewn that the pentagon and trapezium are equal.

On the point H as a centre, with HG as a radius, describe the circular quadrant G*mn*, meeting NH produced in *n*, and making H*n* equal to HG, so that N*n* is equal to the sum of the sides of the rectangular parallelogram GHNO extended in one line. Bisect N*n* in T, and on T as a centre, with N*n* as a diameter, describe the semi-circle *n*KN, meeting GH produced in K; then is HK a mean proportional between HG and HN, the sides of the rectangle GHNO. Consequently, the square, HKLM, described on HK, is equal to the given pentagon ABCDE, for it is equal to the rectangular parallelogram GHNO, and it has been shewn that the pentagon and rectangle are equal.

From the point H draw the straight line HP, making any convenient angle with HF, and set off HP proportional or equal to the number 7·1, or some multiple or submultiple of it, and make HQ equal to HK. Similarly, on HF set off HS proportional or equal to the number 2·26, or the same multiple or submultiple of it that HP is of 7·1, and join PS; then through the point Q, and parallel to PS, draw the straight line QR, meeting HF in the point R, and HR will be the diameter of a circle whose circumference is KH. Bisect KR in I, and on I as a centre with the radius IK or IR, describe the semi circle RWK intersecting H*n* in c; then is cH the radius of a circle, HQW, of equal area as the given pentagon, ABCDE; this is obvious, for by the principles of construction it is equal to the square HKLM, and it has already been shewn, that the pentagon and square are equal. So much then for the conversion. Now for the other operations.

And first, to add pentagons together retaining the form; that is, to constitute a given pentagon that shall be equal in area to two or more given pentagons; or which is the same thing, to convert two or more regular pentagons into a single one containing the same area.

Let it be supposed in the first place, that there are only two given pentagons, and let the straight lines, PQ and RS, fig. 3, denote their sides respectively, PQ being the side of the greater, and RS the side of the lesser. Upon the straight lines PQ and RS as the base and perpendicular, construct the right-angled triangle ABF, right-angled at B, and having AF for its hypothenuse; then, upon the sides AB and BF, which form the legs of the right-angled triangle ABF, constitute by the method shewn in fig. 1, the regular pentagons ABCDE and BFGHI; and in like manner, upon the hypothenuse AF, constitute the regular pentagon AFKLM; then shall this last be equal to the sum of the other two. This is obvious, for by the property of the right-angled triangle, the square of AF is equal to the sum of the squares of AB and BF; but the areas of similar polygons are as the squares of their like sides, therefore, the pentagon AFKLM is equal to both the pentagons ABCDE and BFGHI. If a third pentagon were to be added, it is only necessary to place it in such a manner, that one of its sides shall be at right-angles with a side of the large pentagon just found; then another pentagon similarly constituted upon the hypothenuse, will be the pentagon that is equal to the sum of those given; and so on in the same manner to any extent at pleasure.

In the second place, it is proposed to shew, in what manner one regular pentagon can be subtracted from another, the remainder still retaining the form of a regular pentagon; or, which is the same thing, to convert the difference between two given regular pentagons into a regular pentagon.

Let the straight lines PQ and RS fig. 4, denote the sides of the given pentagons, PQ being the side of the greater, and RS the side of the lesser. Upon AF, which is equal to PQ drawn in any convenient direction, constitute the regular pentagon AMLKF, which will be that from which the other given pentagon is to be taken. At the point F set off on FA and FK respectively, the parts F*b* and F*g*, each equal to RS, the side of the lesser given pentagon, or that which is to be subtracted, and complete the pentagon F*bihg*; then is the intermediate space AMLK*ghib* the difference sought, and that which has to be converted into a pentagon.

Bisect the side AF in O, and on O as a centre with AF as a diameter, describe the semi-circle ABF; then on the point F as a centre, with the distance F*b* as a radius, describe the circular arc *b*B, intersecting the semi-circle ABF in the point B, and draw AB, BF, and ABF is a right-angled triangle, the side AB being that of a pentagon, which is equal to the difference of the given ones, FB being the side of the lesser, or that which is subtracted.

Upon AB and BF construct the pentagons ABCDE and BFGHI, and the first of these will be the pentagon sought, or that which is equal to the difference between the other two, the second being the lesser of the given pentagons, and equal to F*bihg*. The truth of the construction is manifest from the property of the right-angled triangle, combined with that other most elegant and important property, *that the areas of similar polygons, are to one another, as the squares of their homologous sides.*

The multiplication of the pentagon, or the determination of a regular pentagon, that shall be equal in area to any proposed number of times a given pentagon, requires a process somewhat different, but it is nevertheless very easily performed, as follows, on the supposition that the given pentagon is to be multiplied by 3.

Let the straight line RS fig. 5, be the side of the given pentagon, for which one is to be found that will contain its area three times. Draw the straight line IM in any direction, and of any convenient length at pleasure. Assume any point E in the straight line IM, and make EA equal to RS. Upon EA construct the pentagon ABCDE which will be that whose side is given. Make EM equal to three times EA, and upon AM as a diameter and centre K, describe the semi-circle APM intersecting the perpendicular from E in the point P; then is EP the side of the pentagon required. Produce EA and ED the adjacent sides of the pentagon and make EI and EF each equal to EP. Complete the pentagon EFGHI, and it will contain ABCDE three times. This is manifest, for by the principles of construction and the property of the circle, the square of EP, which is equal to EP, is three times the square of EA, and the areas of similar polygons are proportional to the squares of the like sides.

The process of dividing a given pentagon into any proposed number of equal pentagons is very easy, and is effected in the following manner. Let the straight line VW, fig. 6, be the side of the proposed pentagon, which is to be converted into any number of equal pentagons, say three. Draw the straight line IM in any direction, and of any convenient length at pleasure, terminated at I but infinite towards M. From the point I set off towards M the part IE equal to VW the side of the given pentagon, and on IE describe the figure EFGHI, which is that proposed to be divided. At the point E, and in the direction EI, set off ER of any magnitude at pleasure, and make ES equal to three times ER. On RS as a diameter, and Q as a centre, describe the semi-circle RNS, intersecting the perpendicular EP in N, and draw NR, NS. Make EP equal to EF, the side of the given pentagon, and through P draw PA, PM respectively parallel to NR and NS, and meeting IM in the points at A and M; then is EA the side of the pentagon sought. Upon EA describe the pentagon ABCDE, and it will be equal to one-third of the given pentagon EFGHI, as is manifest from the mode of construction itself.

Note.—It may probably be somewhat recommendatory of this method of describing pentagons, to state, that all the figures in this paper have been constructed in this way, and it has been found to apply with almost the same facility and accuracy as the description of a circle with a pair of compasses; it is therefore confidently recommended to practical men, in consequence of its very easy application. T.

THE MAGISTRATES AND THE OFFICIAL REFEREES.

Sir,—Allow me one word more on the subject of magistrates' decisions on the Metropolitan Buildings Act. You advise me, in your note at the foot of my last letter, that I had overlooked the 77th section of the Act. I had not, however, overlooked it, but had, on the contrary, looked it over very carefully, and have done so again, but I cannot bring my mind to see that by that or any other section of the Act, the magistrate is authorized to decide doubts or differences between parties as to the effect of, or the construction to be put upon, any of its provisions, rules, and regulations.

In such a case as Badger *versus* Dean, I conceive that the magistrates' duty is very clear; and it is simply this:—to ascertain whether the work has been done; and if so, the builder's reason for refusing to pay the fees demanded; and upon its being made known to him, that the builder offers a single fee only, whilst the surveyor claims a double fee,—then at once, seeing it to be a question as to the effect of Schedule M., to require of the surveyor to produce the award or certificate of the Official Referees, confirmatory of his claim. This course, I conceive, would be easier for the magistrate, fairer to the parties concerned, more in accordance with the law, and moreover, more beneficial to the public.—I am, Sir, &c.

London, Sept. 7. Δικηγορος.

East Cranmore Church.—The ancient little church of East Cranmore (the door-case of which, still preserved, is supposed to be of Saxon workmanship), having become much dilapidated, was taken down in April, 1845, and an entirely new and enlarged structure, from a design by Mr. T. H. Wyatt, has been erected on the old site. On the 18th of August, the new building, dedicated like the former to St. James, was consecrated by the Lord Bishop of the diocese. *Felix Farley* says, "The church, which is of the early decorated character, with a tower over the porch, surmounted by a broach spire, is substantially built of Doulting freestone, both within and without, and is fitted with spacious open sittings of oak, with an open hammer-beam roof of the same material, supported on corbels of stone, richly carved. The space around the altar is paved with encaustic tiles. The communion-table and rails, reading-desk and pulpit, are of carved oak, with shafts of the Glastonbury pattern."

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